IGNITION BY A HEATED PLATE

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Inzhenerno-Fizicheskii Zhurnal, Vol. 10, No. 4, pp. 523–530, 1966

UDC 536.468

Using the Shvets method, a solution is found to the problem of ignition of a reacting gas by a heated plate, when the gas is at rest and when it is moving due to forced or free convection. Analytical expressions are obtained for the ignition conditions.

We consider a semi-infinite space, full of reacting gas, bounded on the left by a plate whose temperature is held constant $T = T_c$, the initial temperature of the reagent being $T_0 \ll T_c$. We assume that a reaction of zero order occurs, the thermophysical coefficients being constant. We set ourselves the problem of determining the ignition lag for a reacting system of finite dimensions. A similar problem is of interest in the theory of ignition of reacting substances and has been investigated qualitatively in [1, 2], and numerically in another formulation in [3, 4]. Mathematically, the problem reduces to solution of the equation

$$\frac{\partial^2 \theta}{\partial z^2} = \frac{\partial \theta}{\partial \tau} - \exp\theta \tag{1}$$

with the boundary and initial conditions

$$\theta(0, \tau) = 0, \quad \theta(\infty, \tau) = -\theta_0, \quad \theta(z, 0) = -\theta_0. \tag{2}$$

In deriving (1) we used the Frank-Kamenetskii expansion [5] for $\exp(-E/RT)$. Since ignition occurs, as machine calculations for an infinite cylinder have shown [4], in a thin layer at the heated surface, it is appropriate to introduce the thermal boundary layer thickness $\Delta(\tau)$. Then conditions (2) take the form

$$\theta(0, \tau) = 0, \quad \theta(\Delta, \tau) = -\theta_0, \quad \Delta(0) = 0. \tag{3}$$

We shall solve the boundary problem (1)–(3) by the Shvets method [6], which is known for its good convergence and simplicity. Introducing the new independent variable $x = z/\Delta$, and using the results of [7], we may show that for ignition of a layer of reacting substance, $\exp\theta_*$ does not differ much on the average from $\exp(1 - \theta_0 x)$, and so it is natural to choose as a first approximation the profile $\theta_1 = 1 - \theta_0 x$, by substitution of which on the right side of (1) we find, taking account of (3), that

$$\theta_2 = \frac{e\Delta^2}{\theta_0^2}[1 - \exp(-\theta_0 x)] + \frac{\theta_0 x^3 \Delta\dot{\Delta}}{6} - x\left[\theta_0 + \frac{e\Delta^2(1-\gamma)}{\theta_0^2} + \frac{\theta_0\Delta\dot{\Delta}}{6}\right]. \tag{4}$$

By satisfying (4), according to (6), with the condition $\left.\frac{\partial\theta}{\partial x}\right|_{x=1} = 0$, we obtain for $\Delta(\tau)$ a differential equation of the first order, from whose solution, taking into account the last of conditions (3), we have

$$\Delta = \theta_0 \sqrt{\frac{\theta_0}{e(1-\gamma-\gamma\theta_0)}\left\{\exp\left[\frac{6e(1-\gamma-\gamma\theta_0)\tau}{\theta_0^3}\right] - 1\right\}}. \tag{5}$$

In the absence of heat sources [6] $\Delta = 2.45\sqrt{\tau}$. We did not manage to find further analytical approximations for θ in view of the difficulty of integration.

The ignition process may be represented schematically, according to [1, 4], in this way: the reacting mixture is first heated, then a maximum temperature is created, which moves right up to a certain point when the rate of motion of the maximum falls to zero. The time of creation of the maximum temperature corresponds to the end of the heating time, while the time when the rate of motion of the maximum temperature falls to zero is the explosion point. The maximum temperature increases during its motion, and attains very large values by the time of ignition. Analytically the explosion conditions may be written as

$$\left.\frac{\partial\theta}{\partial x}\right|_{x=x_m} = 0, \quad \left.\frac{dx_m}{d\tau}\right|_{\tau=\tau_b} = 0. \tag{6}$$

By satisfying (4) with conditions (6), we obtain a system of two nonlinear equations for Δ and x_m, solution of which gives x_m = const and $\Delta = \infty$. Then we obtain the maximum value $\theta_2(x_m) = \infty$. It is easy to see that $\Delta = \infty$ when $\tau_b = \infty$. Generally speaking, the reacting system is ignited during the last time interval, and $\tau_b = \infty$ is evidence of the crudeness of the second approximation $\theta_2(x, \tau)$. It is important, however, that the quite crude solution (4) correctly reflects the essence of the phenomenon. Since the heating time, according to [4], differs little at large δ from the time to explosion, the condition $\left.\frac{\partial\theta}{\partial x}\right|_{x=0} = 0$, given by Zel'dovich [8], may be considered as an approximate ignition condition which is more accurate, the larger θ_0. By satisfying (4) with the Zel'dovich condition [8], we find $\Delta = \Delta_0$, giving a heating time

$$\Delta_0 = \theta_0 \sqrt{3\theta_0/e[(2+\gamma)\theta_0 - 3(1-\gamma)]}, \tag{7}$$

Variation of $\sqrt{\delta_*}$, $\sqrt{\delta_{*1}}$, Δ_0 and Heating Time τ_h as a Function of θ_0

θ_0	1	2	6	11	18
$\sqrt{\delta_*}$	2.37	2.93	5.46	8.9	13.9
$\sqrt{\delta_{*1}}$	0.90	2.24	9.34	18.2	31.6
Δ_0	1.53	2.29	5.14	8.8	14.0
τ_h	0.23	0.60	3.81	12.0	31.1

and with the aid of (5) and (7) we obtain the heating time

$$\tau_h = \frac{\theta_0^3}{6e(1-\gamma-\gamma\theta_0)} \ln \frac{2\theta_0(1-\gamma)}{[(2+\gamma)\theta_0 - 3(1-\gamma)]}. \quad (8)$$

A reacting system of finite size may evidently be ignited at a given initial temperature T_0, if its dimensionless characteristic size $\sqrt{\delta} > \Delta_0$. Thus, with the help of the solution to the problem of ignition of a semi-infinite reacting space, we may estimate the ignition conditions and the heating time of a reacting substance enclosed between two parallel plates, whose temperatures are T_c and T_0. The table gives values of $\sqrt{\delta_*}$, $\sqrt{\delta_{*1}}$, Δ_0 and τ_h for a number of values of θ_0.

We found the values of $\sqrt{\delta_*}$ with $1 \le \theta_0 \le 6$ using the stationary theory of thermal explosion [5], and values of $\sqrt{\delta_*}$ for $\theta_0 > 6$ were taken from [7]. The quantity $\sqrt{\delta_{*1}}$ was determined from the appropriate formula of [2]. It is interesting to note that, in spite of the crudeness of θ_2, there is good agreement of $\sqrt{\delta_*}$ and Δ_0 at large θ_0, and that $\sqrt{\delta_*}$, $\sqrt{\delta_{*1}}$, Δ_0 increase linearly, in the main, with increase of θ_0.

The problem of ignition of a reacting liquid in a forced convective flow is connected with the problem of flame stabilization by means of smooth surfaces and has been examined in [9, 10]. The problem was solved in [10] for some special cases on a computer, allowing for variation of viscosity and density with temperature. In this paper the problem is solved by the Shvets method [6], using the same values of thermophysical constants as in [10, 11].

Let there be a stream of viscous reacting gas with temperature $T_0 \ll T_c$ flowing over a plate of length l and constant temperature T_c. We shall assume that the flow velocity at ∞ is considerably less than that of sound. We shall neglect burn-up of the reagent, an assumption which, according to [10], does not give a large error, even for second-order reactions. For a given velocity u_∞ we find the plate length for which ignition of the reacting gas occurs. Mathematically, the problem reduces to solution of the system of equations

$$\frac{\partial(\rho v_x)}{\partial x_1} + \frac{\partial(\rho v_y)}{\partial y_1} = 0, \quad (9)$$

$$\rho\left(v_x \frac{\partial v_x}{\partial x_1} + v_y \frac{\partial v_x}{\partial y_1}\right) = \frac{\partial}{\partial y_1}\left(\mu \frac{\partial v_x}{\partial y_1}\right), \quad (10)$$

$$\rho c_p\left(v_x \frac{\partial T}{\partial x_1} + v_y \frac{\partial T}{\partial y_1}\right) = \frac{\partial}{\partial y_1}\left(\lambda \frac{\partial T}{\partial y_1}\right) + q k_0 (c_0 \rho)^n \exp\left(-\frac{E}{RT}\right) \quad (11)$$

with the boundary conditions

$$T(x_1, 0) = T_c, \quad T(x_1, \infty) = T_0, \quad v_x(x_1, 0) = v_y(x_1, 0) = 0, \quad v_x(x_1, \infty) = u_\infty, \quad v_y(x_1, \infty) = 0. \quad (12)$$

We note that frictional heat is neglected in (11), as in (10). Applying the Dorodnitsyn transformation [12] to the system (9)–(11)

$$\bar{y} = \int_0^{y_1} \frac{\rho}{\rho_0} dy_1, \quad \bar{v}_y = \frac{\rho v_y}{\rho_0} + v_x \int_0^{y_1} \frac{\rho}{\rho_0} dy_1, \quad (13)$$

eliminating $\bar{v}_y$, and reducing the system of equations to dimensionless form, taking into account the Frank-Kamenetskii transformation for $\exp(-E/RT)$ [5], we have

$$\frac{\partial^2 u}{\partial y^2} = u \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \int_0^y \frac{\partial u}{\partial x} dy, \quad (14)$$

$$\frac{\partial^2 \theta}{\partial y^2} = P\left(u \frac{\partial \theta}{\partial x} - \frac{\partial \theta}{\partial y} \int_0^y \frac{\partial u}{\partial x} dy\right) - \alpha \exp\theta. \quad (15)$$

The boundary conditions for the system (14)–(15) have the form

$$\theta(x, 0) = 0, \quad \theta(x, \Delta_1) = -\theta_0, \quad u(x, 0) = 0, \quad u(x, \Delta_1) = 1, \quad \Delta_1(0) = 0. \quad (16)$$

We take $\theta_1 = -\theta_0 y/\Delta_1$ as the first approximation for the dimensionless temperature. Substituting θ_1, and the value of u found in [6], in the right side of (15), and integrating the resulting expression, allowing for the first two conditions of (16), we obtain the second approximation

$$\theta_2 = \frac{\alpha\Delta_1^2}{\theta_0^2}\left[1 - \exp\left(-\frac{\theta_0 y}{\Delta_1}\right)\right] + \frac{P\theta_0}{\Delta_1}\left[\frac{y^4 x^{-\frac{1}{2}}}{36}\left(\frac{\dot{\Delta}_1}{\Delta_1} - \frac{1}{4}x^{-1}\right) - \quad (17)$$

$$-\frac{y^7x^{-2}}{32\,256}\left(\frac{\dot{\Delta}_1}{\Delta_1}-\frac{2}{5}x^{-1}\right)-$$

$$-\frac{y}{\Delta_1}\left\{\theta_0+\frac{\alpha\Delta_1^2(1-\gamma)}{\theta_0^2}+P\theta_0\left[\frac{\Delta_1^3x^{-\frac{1}{2}}}{36}\left(\frac{\dot{\Delta}_1}{\Delta_1}-\right.\right.\right.$$

$$\left.\left.\left.-\frac{1}{4}x^{-1}\right)-\frac{\Delta_1^6x^{-2}}{32\,256}\left(\frac{\dot{\Delta}_1}{\Delta_1}-\frac{2}{5}x^{-1}\right)\right]\right\}. \quad (17)\ \text{cont'd}$$

Following [6], we obtain the differential equation for the determination of Δ_1,

$$Px\left(\frac{x^{3/2}}{36}-\frac{w}{16128}\right)\frac{dw}{dx}-Pw\left(\frac{x^{3/2}}{48}-\right.$$

$$\left.-\frac{w}{13440}\right)=x^3\left[1+\frac{\alpha(1-\gamma-\gamma\theta_0)w^{2/3}}{\theta_0^3}\right]. \quad (18)$$

If E = ∞, $\alpha = 0$, and we have an equation for the thermal boundary layer in the absence of reactions [6]. Dropping the terms in (18) with small coefficients 1/16128, 1/13440, $1/\theta_0^3$, and solving the resulting linear differential equation, we find

$$\Delta_1 \approx (48/P)^{1/3}\sqrt{x}. \quad (19)$$

Expression (19) agrees with the corresponding expression of [6], within the limits of the approximations made. This is in agreement with the results of [10], according to which the boundary layer thickness, with the reaction taken into account, differs little from the thickness of the thermal boundary layer when reactions up to the ignition point are not allowed for. A more exact investigation, taking the discarded terms into account, indicates, in agreement with [10], that in the absence of reaction ($\alpha = 0$) the boundary layer is thinner than the thermal boundary layer when heat release from the reaction ($\alpha \neq 0$) is allowed for. This is easily verified by applying Chaplygin's theorem concerning differential inequalities to (18), allowing for the last of conditions (16). The solution (19) could be improved according to the method of [13], but such improvement would not be worthwhile, since there is substantial error in the measurement of E. As the ignition condition we shall take, as before, the Zel'dovich [8] condition, which in this case means that the heat flux from the heated plate equals zero at a certain $x = x_0$, i.e., from $x = x_0$ onwards, the plate is not heating the gas, but, on the contrary, the reacting gas is heating the plate, owing to the heat released in the reaction. Satisfying (17) with the ignition condition $\left.\frac{\partial\theta}{\partial y}\right|_{y=0}=0$, and solving the resulting equation and (18) relative to Δ_1, we have, taking (19) into account,

$$x_0 = 0.\,00126189(80P-3)\,\theta_0^2/\alpha P^{1/3}. \quad (20)$$

Using (20), we can find the value of l corresponding to ignition of the reacting mixture at a given point on the plate. Knowing l, we can estimate the dwell time $t_* = lx_0/u_\infty$ of a liquid particle required for ignition. If $x_0 > 1$, ignition of the reagent does not occur on the plate, i.e., (20) can serve as the ignition condition. Comparison of (20) with the corresponding formula of [8] has shown that the two are qualitatively equivalent and much the same quantitatively. The ignition condition will be more accurate if the Frank-Kamenetskii transformation for exp(E/RT) [5] is not used. For a first-order reaction we may similarly obtain

$$lx_0 = [0.00126189(80P-3)\,c_pu_\infty(T_c-T_0)^2]\times$$

$$\times\left[qk_0c_0T_0P^{4/3}\left\{\left[Ei\left(-\frac{E}{RT_c}\right)-\right.\right.\right.$$

$$\left.-Ei\left(-\frac{E}{RT_0}\right)\right]\frac{E}{RT_0}-\exp\left(-\frac{E}{RT_0}\right)+$$

$$\left.\left.+\frac{T_c}{T_0}\exp\left(-\frac{E}{RT_c}\right)\right\}\right]^{-1}, \quad (21)$$

and for a second-order reaction we have, correspondingly,

$$lx_0 = \frac{0.00126189(80P-3)\,c_pu_\infty(T_c-T_0)^2}{qc_0^2\rho_0k_0P^{4/3}T_0[Ei(-E/RT_0)-Ei(-E/RT_c)]}. \quad (22)$$

Values of x_0 found from (22) agree in order of magnitude with the numerical results of [10]. There is no difficulty, in principle, in examining ignition by the Shvets method [6] allowing for variation of reagent concentration, but the computations are onerous and the final formulas unwieldy.

In practice any ignition process is connected with free convection of the reacting substance. We shall examine ignition by a heated vertical plate at temperature T_c washed by a viscous, incompressible reacting liquid, whose temperature is $T_0 \ll T_c$. We assume that the thermophysical properties are constant, and that a zeroth-order reaction occurs. Mathematically, the problem reduces to solution of the system of equations

$$\frac{\partial v_x}{\partial x_1}+\frac{\partial v_y}{\partial y_1}=0, \quad (23)$$

$$v_x\frac{\partial v_x}{\partial x_1}+v_y\frac{\partial v_x}{\partial y_1}=\nu\frac{\partial^2 v_x}{\partial y_1^2}+g\beta(T-T_0), \quad (24)$$

$$\rho_0c_p\left(v_x\frac{\partial T}{\partial x_1}+v_y\frac{\partial T}{\partial y_1}\right)=$$

$$=\lambda\frac{\partial^2 T}{\partial y_1^2}+qk_0\exp(-E/RT) \quad (25)$$

with boundary conditions

$$T(x_1,0)=T_c,\ T(x_1,\infty)=T_0,\ v_x(x_1,0)=$$

$$=v_y(x_1,0)=0,\ v_x(x_1,\infty)=v_y(x_1,\infty)=0. \quad (26)$$

The coordinate origin is located at the lower edge of the plate, the x axis being along the plate, and the y axis perpendicular to it. Eliminating v_y from the system (23)–(25) and reducing the system to dimensionless form, taking into account the Frank-Kamenetskii transformation [5] for exp(−E/RT), we obtain

$$\frac{\partial^2 U}{\partial \eta^2} = \left(U \frac{\partial U}{\partial \xi} - \frac{\partial U}{\partial \eta} \int_0^\eta \frac{\partial U}{\partial \xi} d\eta\right) - \theta_0 - \theta, \quad (27)$$

$$\frac{\partial^2 \theta}{\partial \eta^2} = P\left(U \frac{\partial \theta}{\partial \xi} - \frac{\partial \theta}{\partial \eta} \int_0^\eta \frac{\partial U}{\partial \xi} d\eta\right) - a \exp \theta. \quad (28)$$

The boundary conditions for the system of equations (26), (27) have the form

$$\theta(\xi,\ 0) = 0, \quad \theta(\xi,\ \Delta_2) = -\theta_0,$$
$$U(\xi,\ 0) = U(\xi,\ \Delta_2) = 0, \quad \Delta_2(0) = 0. \quad (29)$$

We take the first approximations for θ and U in the form

$$\theta_1 = -\theta_0 \eta/\Delta_2, \quad U_1 = \theta_0 (\eta^2/6\,\Delta_2 - \eta/2 + \Delta_2/3)\,\eta. \quad (30)$$

Substituting (30) into the right side of (28) and integrating the result, taking account of boundary conditions (29), we obtain the second approximation

$$\theta_2 = \frac{a\,\Delta_2^2}{\theta_0^2}\left[1 - \exp\left(-\frac{\theta_0 \eta}{\Delta_2}\right)\right] + \frac{P\,\theta_0^2\,\dot{\Delta}_2 \eta^4}{\Delta_2}\left(\frac{\eta^2}{240\,\Delta_2^2} - \frac{\eta}{40\,\Delta_2} + \frac{1}{24}\right) - \frac{\eta}{\Delta_2}\left[\theta_0 + \frac{a\,\Delta_2}{\theta_0^2}(1-\gamma) + \frac{P\,\theta_0^2\,\dot{\Delta}_2\,\Delta_2^3}{48}\right]. \quad (31)$$

Following [6], we get the differential equation for determining $\Delta_2(\xi)$:

$$\frac{11 P\,\theta_0^2}{240}\,\Delta_2^3\,\frac{d\Delta_2}{d\xi} = \theta_0 + \frac{a(1-\gamma-\gamma\theta_0)}{\theta_0^2}\,\Delta_2^2. \quad (32)$$

Integrating (32), and taking into account the last of conditions (29), we have

$$\xi = \frac{11 P\,\theta_0^7}{480 a^2 (1-\gamma-\gamma\theta_0)^2}\left\{\frac{a\,\Delta_2^2(1-\gamma-\gamma\theta_0)}{\theta_0^3} - \ln\left[1 + \frac{a\,\Delta_2^2(1-\gamma-\gamma\theta_0)}{\theta_0^3}\right]\right\}. \quad (33)$$

Satisfying the ignition condition $\left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0} = 0$, we obtain

$$\frac{P\,\theta_0^2}{48}\,\Delta_2^3\,\frac{d\Delta_2}{d\xi} = \frac{a\,\Delta_2^2}{\theta_0^2}(\theta_0 - 1 + \gamma) - \theta_0. \quad (34)$$

Eliminating Δ_2 from (32) and (34), we find the quantity $\Delta_2 = \Delta_{02}$, corresponding to the ignition condition,

$$\Delta_{02} = 16\,\theta_0^3/a(11\theta_0 - 16 + 16\gamma + 5\gamma\theta_0). \quad (35)$$

Substituting (35) into (33), we obtain the quantity $\xi = \xi_0$ corresponding to ignition

$$\xi_0 = \frac{11 P\,\theta_0^7}{480 a^2 (1-\gamma-\gamma\theta_0)^2}\left\{\frac{16(1-\gamma-\gamma\theta_0)}{11\theta_0 - 16 + 16\gamma + 5\gamma\theta_0} - \ln\left[1 + \frac{16(1-\gamma-\gamma\theta_0)}{11\theta_0 - 16 + 16\gamma + 5\gamma\theta_0}\right]\right\}. \quad (36)$$

Knowing T_c, T_0, and the physical and kinetic constants, we can easily find, using (36), the dimensionless distance from the edge of the plate at which ignition of the reacting liquid takes place. For a nonreacting liquid E = ∞, and it follows from (36) that $\xi_0 = \infty$, i.e., in this case there is no ignition. Therefore, if the dimensionless length of the plate is greater than ξ_0, ignition occurs, while in the opposite case there is no ignition of the reacting liquid on the plate surface.

NOTATION

$\theta = (T - T_c)\,E/RT_c^2$—dimensionless temperature; E—activation energy; R—universal gas constant; T_c—temperature of heated plate; q—thermal effect of reaction, $-\theta_0 = (T_0 - T_c)\,E/RT_c^2$—dimensionless initial temperature and temperature of reacting liquid outside boundary layer; T_0—initial temperature of liquid and temperature outside boundary layer; $x = x_1/l$, $y = \frac{\sqrt{Re}}{l}\int_0^{y_1}\frac{\rho}{\rho_0}\,dy_1$, $z = x_1\sqrt{\frac{k_0 E}{\lambda R T_c^2}\exp\left(-\frac{E}{RT_c}\right)}$, $\xi = x_1 (g\beta RT_c^2/E\nu^2)^{1/3}$, $\eta = y_1 (g\beta RT_c^2/E\nu^2)^{1/3}$—dimensionless coordinates; l—characteristic dimension; x_1, y_1—dimensional coordinates; k_0—preexponent; λ—thermal conductivity; $\sqrt{\delta} = \frac{l}{T_c}\left(\frac{qEk_0}{\lambda R}\cdot\exp\left(-\frac{E}{RT_c}\right)\right)^{1/2}$—dimensionless characteristic dimension in [7]; $\sqrt{\delta_*}$—critical value of $\sqrt{\delta}$, at which a real solution of equation (2i) of [7] still exists; θ_*—limiting real solution of (2i) of [7]; Re—Reynolds number; Δ, Δ_1, Δ_2—dimensionless thermal boundary layer thickness for a liquid at rest, forced convection, and free convection, respectively; $\tau = \frac{qtEk_0}{c_p \rho_0 RT_c^2}\exp\left(-\frac{E}{RT_c}\right)$—dimensionless time; t—time; τ_h—heating time; τ_b—ignition time lag; c_p—specific heat at constant pressure; ρ_0—density at $T = T_0$; P—Prandtl number; $a = \frac{qk_0 c_0 P l E}{u_\infty c_p R T_c^2}\cdot\exp\left(-\frac{E}{RT_c}\right)$—dimensionless plate length; $w = \Delta_1^3$; $\gamma = \exp(-\theta_0)$; $u = \frac{v_x}{u_\infty}$, $v = \frac{v_y\sqrt{Re}}{u_\infty}$, $U = v_x\left(\frac{E}{g\beta\nu RT_c^2}\right)^{1/3}$, $V = v_y\left(\frac{E}{g\beta\nu RT_c^2}\right)^{1/3}$ dimensionless longitudinal and transverse flow velocity components for forced and free convection, respectively; c_0—initial concentration; β—volume expansion coefficient; $a = \frac{qk_0 E}{\lambda RT_c^2}\left(\frac{E\nu^2}{g\beta RT_c^2}\right)^{2/3}\cdot\exp\left(-\frac{E}{RT_c}\right)$—dimensionless parameter; $\dot{\Delta} = \frac{d\Delta}{d\tau}$; $\dot{\Delta}_1 = \frac{d\Delta_1}{dx}$; $\dot{\Delta}_2 = \frac{d\Delta_2}{d\xi}$; x_0, ξ_0—dimensionless distance from edge of heated plate at which reacting liquid ignites, for free and forced convection, respectively; g—acceleration due to gravity; n—order of reaction; μ—viscosity; ν—kinematic viscosity at $T = T_0$.

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23 August 1965

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